

Centre of Mass, cm (or centre of gravity, cg) and Torque,  $\tau$ .

Leaning tower of Pisa.



not balanced

Treat  $F_g$  as acting  
at the centre of mass.  
Distance to rotation  
point,  $r$ , creates  
a torque,  $\tau$

$$\tau = F \times r$$

$\tau$  vector  
vector  
cross  
product

vector

$$\tau = Fr \sin \theta$$

$\theta$  is angle between  $F$  and  $r$ .  
units Nm (not J)

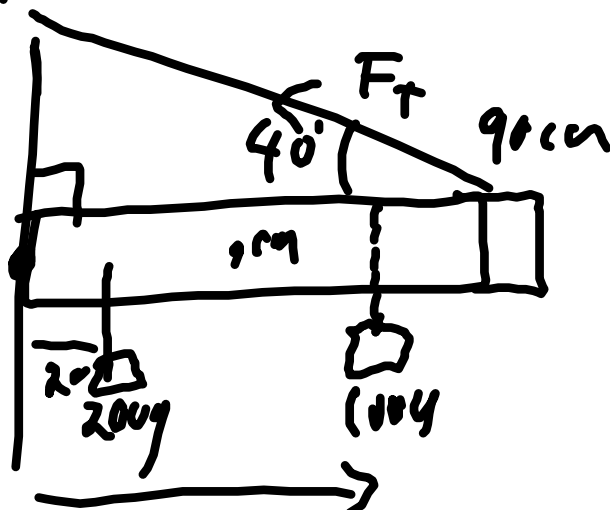
Rotational Equilibrium - the torques on an object balance  
torques clockwise = torques counterclockwise

$$\tau_c = \tau_{cc}$$

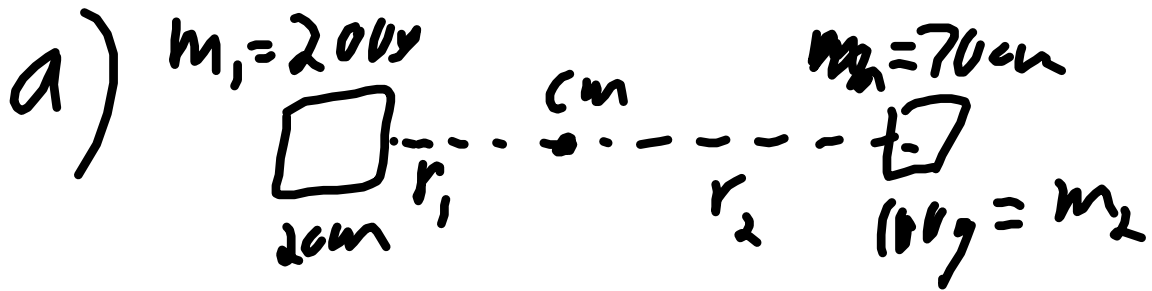
e.g. You hang a 200g mass on a massless meter stick at 20cm and a 100g mass at 100cm.

at 70cm, 100g mass

- a) Where is the centre of mass?
- b) if the meter stick is 150g, where is the cm?
- c) you secure the meter stick to the wall at 0cm and with a string at 90cm making a  $40^\circ$  angle to the stick. What is the tension in the string?



$$\xrightarrow{20}$$



$$r_1 + r_2 = 50 \text{ cm}$$

$\tau_{cc}$   $m_1 g r_1 = m_2 g r_2$   $\tau_c$

$$r_1 = \left( \frac{m_2}{m_1} r_2 \right)$$

$$\frac{m_2}{m_1} r_2 + r_2 = r_T$$

$$\left( \frac{m_2}{m_1} + 1 \right) r_2 = r_T$$

$$r_2 = \frac{50}{\frac{70}{200} + 1}$$

$$r_2 = 33.3 \text{ cm}$$

$$C_m = 70 \text{ cm} - 33.3 \text{ cm} \\ = \boxed{36.7 \text{ cm}}$$

$$\sum M x_{cm} = m_1 x_1 + m_2 x_2 \dots$$

general equation

b) Since metre stick is  
150g and uniform,  
Set mass at centre, 50cm.

$$(100\text{g} + 200\text{g} + 150\text{g}) x_{cm} = 200\text{g}(20\text{cm}) + 100\text{g}(-20\text{cm}) \\ + 150\text{g}(50\text{cm})$$

$$\boxed{x_{cm} = 41 \text{ cm}}$$